Persistent Observables: A generalization of the gravitational wave memory effect

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[I]	arXiv:1901.00021	w/ É. Flanagan, A. Harte, D. Nichols
[II]	arXiv:1912.13449	"
[III]	arXiv:2109.03832	w/ D. Nichols

Overview

- Gravitational wave (displacement) memory: change in displacement of initially comoving observers $\gamma, \bar{\gamma}$ [Zel'dovich & Polnarev, 1974]:

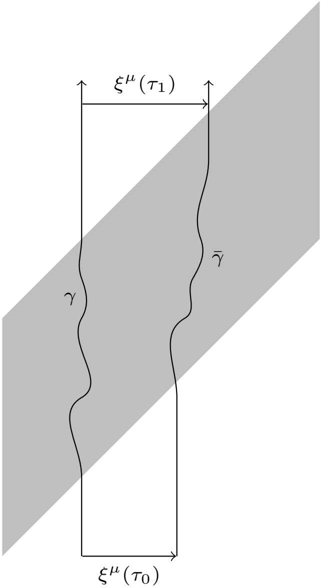
$$\Delta\xi^\alpha \sim \iint R^\alpha{}_{\mu\beta\nu}\dot{\gamma}^\mu\dot{\gamma}^\nu\xi^\beta$$

- *Not* the only permanent effect: "persistent observables" provide general framework

- Generation of displacement memory:

$$\text{Displacement memory} = \Delta(\text{source properties}) + \int(\text{flux of gravitational waves})$$

- "Curve deviation" observable obeys similar formula



Outline

I. Persistent observables

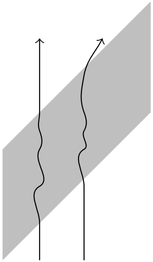
II. The curve deviation observable

III. Asymptotic properties

IV. Relation to “charges” and “fluxes”

"New memory effects"

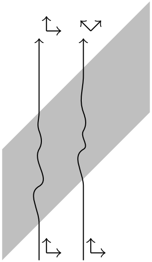
Usual "displacement" memory is not the only permanent effect:



Velocity

[Grishchuk & Polnarev, 1989]

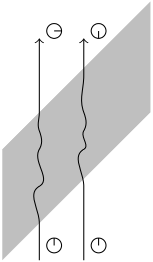
$$\sim \int R^{\mu}{}_{\alpha\nu\beta}\dot{\gamma}^{\alpha}\dot{\gamma}^{\beta}\xi^{\nu}$$



Rotation

[Flanagan & Nichols, 2014]

$$\sim \int R^{\mu}{}_{\nu\alpha\beta}\xi^{\alpha}\dot{\gamma}^{\beta}$$



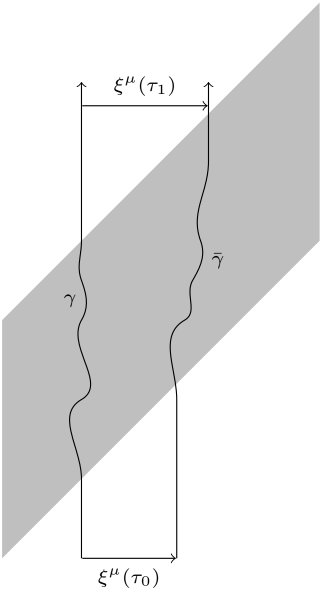
Proper time shift

[Strominger & Zhiboedov, 2014]

$$\sim \int R_{\alpha\beta\gamma\delta}\dot{\gamma}^{\alpha}\xi^{\beta}\dot{\gamma}^{\gamma}\xi^{\delta}$$

Persistent observables

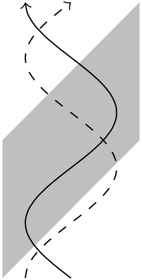
- Measurement over a time interval that vanishes in absence of radiation
- "Absence of radiation" requires regime where radiation is even defined:
  - Linearized solutions off fixed background
  - Asymptotically flat spacetimes
  - Nonlinear plane wave spacetimes
  - etc.
- Classification focuses on *how they are observed*; different physical effects can be equal at leading order!



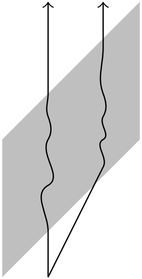
Example of "degeneracy": spin memory

Two notions of spin memory, both $\sim \iiint R^\mu{}_{\alpha\nu\beta}\dot\gamma^\alpha\dot\gamma^\beta\xi^\nu$:

- Phase shift in Sagnac interferometer [Pasterski et al., 2015]
- Local, but $O(1/r^2)$



- Magnetic part of "subleading displacement" [Nichols, 2017], [I]
- Nonlocal, but $O(1/r)$

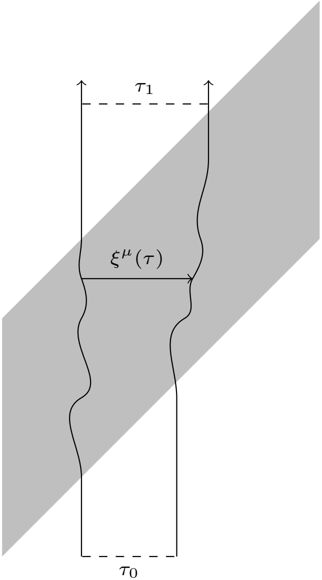


Effective measurement

- GW detectors measure only one physical observable: (instantaneous) separation/phase shifts
- However, give ξ^α over time
  $\implies$ measure (certain components of) $R^\alpha{}_{\beta\gamma\delta}$:

$$\xi^\alpha \sim \iint R^\alpha{}_{\mu\beta\nu}\dot{\gamma}^\mu\dot{\gamma}^\nu\xi^\beta \implies R^\alpha{}_{\mu\beta\nu}\dot{\gamma}^\mu\dot{\gamma}^\nu\xi^\beta$$

- Weak-field limit of (many) persistent observables computable from integrals of $R^\alpha{}_{\beta\gamma\delta}$
- Practical way to "measure" these observables



Summary of persistent observables (not complete)

Observable	Integrals of Riemann tensor	Leading order as $r \to \infty$	Reference	
Displacement	2	$1/r$	[Zeldovich & Polnarev, 1974]	Curve dev.; Ang. mom. holo.
Subleading displacement	3	$1/r$	[Nichols; 2017, 2018]	
Proper time shift	1	$1/r^2$	[Strominger & Zhiboedov, 2014]	
Velocity	1	$1/r^2$	[Grishchuk & Polnarev, 1989]	Holonomy
"Subleading" velocity (kick)	2	$1/r$	[Seraj, 2021], [II] (briefly)	
Radial kick	1	$1/r^3$	[Godazgar et al., 2022]	
Rotation	1	$1/r^2$	[Flanagan & Nichols, 2014]	
"Original" spin memory	1?	$1/r^2$	[Pasterski et al., 2015]	Holo.? Ali's talk
Gyroscopic memory	1?	$1/r^2$	[Seraj & Oblak, 2021]	
Curve deviation	$\geq 1^a$	$1/r$	[I]	⟵ **This talk**
Angular momentum holonomy	$\geq 1^a$	$1/r$	[I]	
Spinning test particle observables	1–2	$1/r$, $1/r^2$?	[I]	

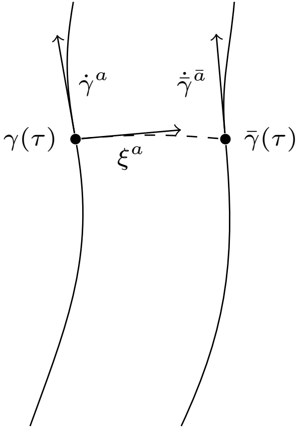
[a] Only ≥ 4 if there is acceleration.

Outline

Geodesic deviation

- Two observers, following γ and $\bar{\gamma}$, w/ four-velocities $\dot{\gamma}^a$ and $\dot{\bar{\gamma}}^{\bar{a}}$
- Separation vector ξ^a tangent to unique geodesic between $\gamma(\tau)$ and $\bar{\gamma}(\tau)$
- Geodesic deviation equation:

$$\ddot{\xi}^a = -\underbrace{R^a{}_{cbd}\dot{\gamma}^c\dot{\gamma}^d}_{\equiv R^a{}_{\dot{\gamma}b\dot{\gamma}}}\xi^b + O(\boldsymbol{\xi}, \dot{\boldsymbol{\xi}})^2$$



For an excellent review, see [Vines, 2014]

Displacement memory

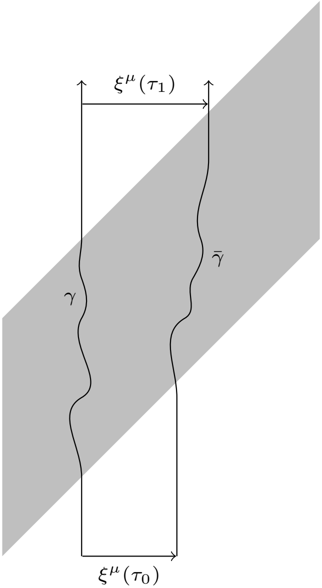
- Note: *tensor* equations cannot be integrated directly!
- To solve, write *parallel-transported* tetrad $\{\boldsymbol{e}_\alpha\}$:

$$\frac{\mathrm{D}\boldsymbol{e}_\alpha}{\mathrm{d}\tau} = 0 \implies \ddot{\xi}^\alpha(\tau) = -R^\alpha{}_{\dot{\gamma}\beta\dot{\gamma}}(\tau)\xi^\beta(\tau) + O(\boldsymbol{\xi}, \dot{\boldsymbol{\xi}})^2$$

- Integration of 2nd order ODE: for $\dot{\xi}^\alpha(\tau_0) = 0$,

$$\xi^\alpha(\tau_1) = \xi^\alpha(\tau_0) + \Delta K^\alpha{}_\beta(\tau_1, \tau_0)\xi^\beta(\tau_0) + O(\boldsymbol{\xi}, \dot{\boldsymbol{\xi}})^2, \text{ where}$$

$$\Delta K^\alpha{}_\beta(\tau_1, \tau_0) \equiv -\int_{\tau_0}^{\tau_1} \mathrm{d}\tau_2 \int_{\tau_0}^{\tau_2} \mathrm{d}\tau_3 \, R^\alpha{}_{\dot{\gamma}\beta\dot{\gamma}}(\tau_3) + O(\boldsymbol{R}^2)$$



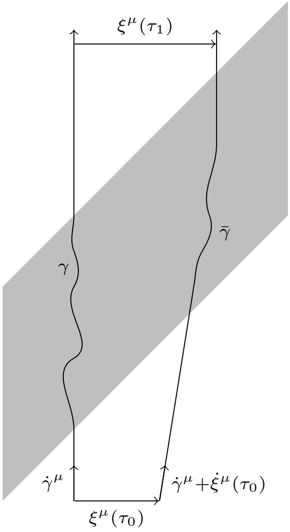
Subleading displacement memory

- Solution when $\dot{\xi}^\alpha(\tau_0) \neq 0$:

$$\xi^\alpha(\tau_1) = \xi^\alpha(\tau_0) + \Delta K^\alpha{}_\beta(\tau_1, \tau_0)\xi^\beta(\tau_0) + (\tau_1 - \tau_0)\left[\dot{\xi}^\alpha(\tau_0) + \Delta H^\alpha{}_\beta(\tau_1, \tau_0)\dot{\xi}^\beta(\tau_0)\right] + O(\boldsymbol{\xi}, \dot{\boldsymbol{\xi}})^2, \text{ where}$$

$$(\tau_1 - \tau_0)\Delta H^\alpha{}_\beta(\tau_1, \tau_0) \equiv -\int_{\tau_0}^{\tau_1} \mathrm{d}\tau_2 \int_{\tau_0}^{\tau_2} \mathrm{d}\tau_3 \, (\tau_3 - \tau_0) R^\alpha{}_{\dot{\gamma}\beta\dot{\gamma}}(\tau_3) + O(\boldsymbol{R}^2)$$

- E & B decomposition related to CoM [Nichols, 2018] & spin [Nichols, 2017] memories



The addition of acceleration: "Non-geodesic" deviation

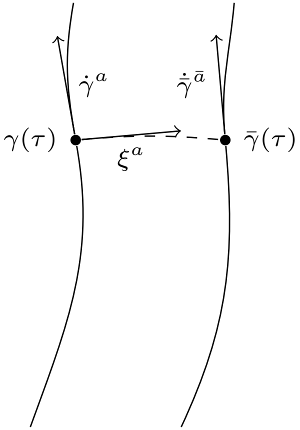
If $\bar{\gamma}$ accelerated, geodesic deviation modified:

$$\ddot{\xi}^a = -R^a{}_{\dot{\gamma}b\dot{\gamma}}\xi^b + a^a + O(\boldsymbol{\xi}, \dot{\boldsymbol{\xi}}, \boldsymbol{a})^2,$$

where a^a measures the "relative acceleration":

$$a^a \equiv g^a{}_{\bar{a}}\ddot{\bar{\gamma}}^{\bar{a}},$$

where $g^a{}_{\bar{a}}$ maps $\bar{\gamma}(\tau) \to \gamma(\tau)$ by parallel transport



"Kinematic" flat-space contribution

- On our tetrad:

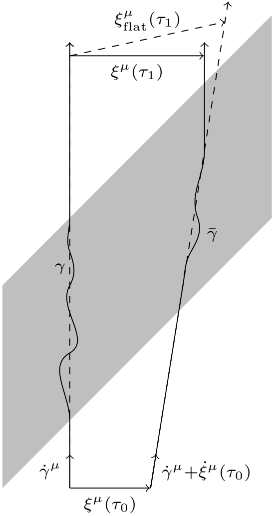
$$\ddot{\xi}^\alpha(\tau) = -R^\alpha{}_{\dot{\gamma}\beta\dot{\gamma}}(\tau)\xi^\beta(\tau) + a^\alpha(\tau) + O(\boldsymbol{\xi}, \dot{\boldsymbol{\xi}}, \boldsymbol{a})^2$$

- Solution has contribution even in flat space:

$$\ddot{\xi}^\alpha_{\text{flat}}(\tau) = a^\alpha(\tau)$$

$$\Downarrow$$

$$\xi^\alpha_{\text{flat}}(\tau_1) = \xi^\alpha(\tau_0) + (\tau_1 - \tau_0)\dot{\xi}^\alpha(\tau_0) + \int_{\tau_0}^{\tau_1} \mathrm{d}\tau_2 (\tau_1 - \tau_2) a^\alpha(\tau_2)$$



Curve deviation

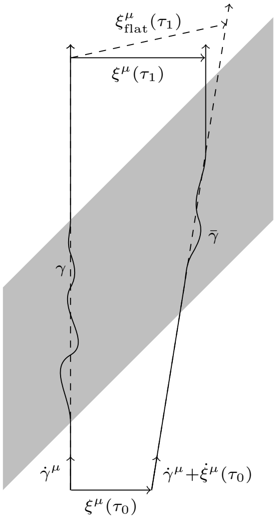
- Remove flat space contribution to get *curve deviation observable*:

$$\Delta\xi^\alpha(\tau_1,\tau_0) \equiv \xi^\alpha(\tau_1) - \xi^\alpha_{\text{flat}}(\tau_1)$$

- Parameterize in terms of initial data:

$$\begin{aligned}\Delta\xi^\alpha(\tau_1,\tau_0) \equiv\ & \Delta K^\alpha{}_\beta(\tau_1,\tau_0)\xi^\beta(\tau_0) \\ & + (\tau_1-\tau_0)\Delta H^\alpha{}_\beta(\tau_1,\tau_0)\dot{\xi}^\beta(\tau_0) \\ & + \sum_{n=0}^{\infty} \Delta \underset{(n)}{\alpha}{}^\alpha{}_\beta(\tau_1,\tau_0)\left.\frac{\mathrm{D}^n a^\alpha}{\mathrm{d}\tau^n}\right|_{\tau=\tau_0}\end{aligned}$$

- Yields old and new observables



Leading order in curvature

- Curvature is typically weak, so it makes sense to expand these solutions:

$$\begin{aligned}
\Delta K^\alpha{}_\beta(\tau_1, \tau_0) &= -\int_{\tau_0}^{\tau_1} \mathrm{d}\tau_2 \int_{\tau_0}^{\tau_2} \mathrm{d}\tau_3 R^\alpha{}_{\dot\gamma\beta\dot\gamma}(\tau_3) + O(\boldsymbol{R}^2), \\
(\tau_1 - \tau_0)\Delta H^\alpha{}_\beta(\tau_1, \tau_0) &= -\int_{\tau_0}^{\tau_1} \mathrm{d}\tau_2 \int_{\tau_0}^{\tau_2} \mathrm{d}\tau_3 (\tau_3 - \tau_0) R^\alpha{}_{\dot\gamma\beta\dot\gamma}(\tau_3) + O(\boldsymbol{R}^2), \\
\Delta \underset{(n)}{\alpha}{}^\alpha{}_\beta(\tau_1, \tau_0) &= -\frac{1}{n!}\int_{\tau_0}^{\tau_1} \mathrm{d}\tau_2 (\tau_2 - \tau_0)^n \int_{\tau_2}^{\tau_1} \mathrm{d}\tau_3 \int_{\tau_2}^{\tau_3} \mathrm{d}\tau_4 (\tau_4 - \tau_2) R^\alpha{}_{\dot\gamma\beta\dot\gamma}(\tau_4) \\
&\quad + O(\boldsymbol{R}^2).
\end{aligned}$$

- Dependence on higher derivatives of $\boldsymbol{\xi}$ $\Leftrightarrow$ higher integrals of $\boldsymbol{R}$

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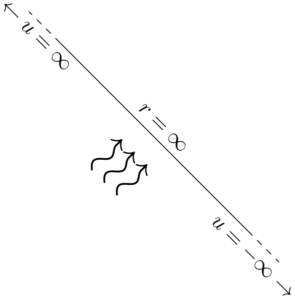
Bondi(-Sachs) coordinates

- Key parts of metric:

$$g_{uu} = -1 + \frac{2m}{r} + O(1/r^2),$$

$$g_{ui} = -\frac{1}{2}\mathscr{D}^j C_{ij} + \frac{1}{r}\left(\frac{2}{3}N_i + \cdots\right) + O(1/r^2),$$

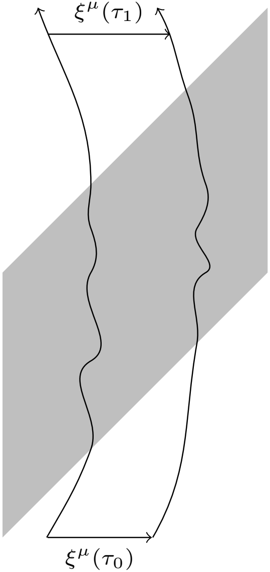
$$g_{ij} = r^2\left\{[1 + O(1/r^2)]h_{ij} + \frac{1}{r}\left(C_{ij} + \frac{1}{r^2}\sum_{n=0}^{\infty}\frac{1}{r^n}\underset{(n)}{\mathcal{E}}{}_{ij}\right)\right\}$$



- *Shear* C_{ij} (waveform), unnamed $\underset{(n)}{\mathcal{E}}{}_{ij}$ [$(n+2)$-pole aspect?]
- m, N^i: *mass* and *angular momentum aspect*
- $N_{ij} = \partial_u C_{ij}$: *news*, indicates presence of radiation
- m, N^i, $\underset{(n)}{\mathcal{E}}{}_{ij}$: properties of source
  (essentially $\text{Re}[\psi_2]$, ψ_1, ψ_0^n)

A note on asymptotic observables

- In [I], only considered *flat-to-flat* transitions
  $\implies$ persistent observables involve integrals of *curvature*
- Situation now *nonradiative-to-nonradiative*
  $\implies$ "true" persistent observables involve integrals of N_{ij}
- At high orders, observables measure more than radiation
- Conjecture: do not look at observables beyond $O(1/r^2)$
- Will focus on $O(1/r)$ observables
- Related note: assuming cutoff for N_{ij} (not realistic)



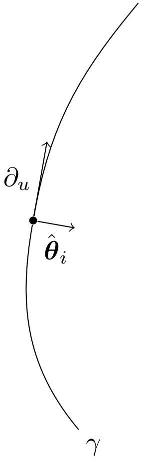
Asymptotic observers

- We consider an observer with four-velocity

$$\dot{\gamma}^a = (\partial_u)^a + O(1/r),$$

which gives an *approximate* geodesic

- Proper time $\tau \to$ retarded time u
- Similarly, $(\hat{\theta}_i)^a \equiv \frac{1}{r}(\partial_i)^a$, $(\hat{\theta}^i)_a \equiv r(\mathrm{d}\theta^i)_a$ *approximately* parallel-transported along γ
- What are the curve deviation observables to leading order?



Asymptotic form

- Substitute $\tau \to u$ & $\dot{\gamma}^a = (\partial_u)^a$, compute components along $\{\hat{\boldsymbol{\theta}}_i\}$, & substitute $R^i{}_{uju} = -\frac{1}{2r}\dot{N}^i{}_j + O(1/r^2)$:

$$\begin{aligned}
\Delta K^i{}_j(u_1, u_0) &= \frac{1}{2r}\int_{u_0}^{u_1} \mathrm{d}u_2 \int_{u_0}^{u_2} \mathrm{d}u_3 \dot{N}^i{}_j(u_3) + O(1/r^2), \\
(u_1 - u_0)\Delta H^i{}_j(u_1, u_0) &= \frac{1}{2r}\int_{u_0}^{u_1} \mathrm{d}u_2 \int_{u_0}^{u_2} \mathrm{d}u_3 (u_3 - u_0)\dot{N}^i{}_j(u_3) + O(1/r^2), \\
\Delta \underset{(n)}{\alpha}{}^i{}_j(u_1, u_0) &= \frac{1}{2r}\int_{u_0}^{u_1} \mathrm{d}u_2 \frac{(u_2 - u_0)^n}{n!} \int_{u_2}^{u_1} \mathrm{d}u_3 \int_{u_2}^{u_3} \mathrm{d}u_4 (u_4 - u_2)\dot{N}^i{}_j(u_4) \\
&\quad + O(1/r^2).
\end{aligned}$$

Moments of the news

- *Moments* provide useful way of characterizing multiple integrals:

$$\underset{(n)}{\mathcal{N}}{}^i{}_j(u_1, u_0) \equiv \frac{1}{n!} \int_{u_0}^{u_1} \mathrm{d}u_2\, (u_2 - u_0)^n N^i{}_j(u_2)$$

- By integrations by parts (dropping boundary terms—assumes news cuts off),

$$\Delta K^i{}_j(u_1, u_0) = \frac{1}{2r} \underset{(0)}{\mathcal{N}}{}^i{}_j(u_1, u_0) + O(1/r^2),$$

$$(u_1 - u_0)\Delta H^i{}_j(u_1, u_0) = \frac{1}{2r}\left[2\underset{(1)}{\mathcal{N}}{}^i{}_j(u_1, u_0) - (u_1 - u_0)\underset{(0)}{\mathcal{N}}{}^i{}_j(u_1, u_0)\right] + O(1/r^2),$$

$$\Delta \underset{(n)}{\alpha}{}^i{}_j(u_1, u_0) = \frac{1}{2r}\left[(n+3)\underset{(n+2)}{\mathcal{N}}{}^i{}_j(u_1, u_0) - (u_1 - u_0)\underset{(n+1)}{\mathcal{N}}{}^i{}_j(u_1, u_0)\right] + O(1/r^2)$$

- Remainder of talk: *only* concerned with moments

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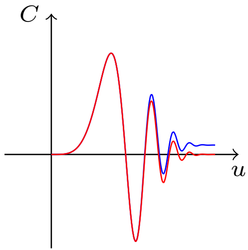
Computation of the moments

- Given a full waveform C_{ij}, one can easily compute these moments by integration
- Unfortunately, we do not have a "full" waveform:
  - PN waveforms only valid up to some order in v/c
  - Numerical waveforms *extrapolate* signals at finite radius to infinity (CCE fixes this—see Keefe's talk)

$\implies$ Moments computed directly are inaccurate!

- Fortunately, relationships exist involving these moments, providing a *consistency check*



Mass aspect and the zeroth moment

- Evolution of m (from $R_{uu} = 0$):

$$\partial_u m = \frac{1}{4}\mathscr{D}_i\mathscr{D}_j N^{ij} \underbrace{-\frac{1}{8}N_{ij}N^{ij}}_{\equiv \mathcal{F}}$$

- Integrate to get zeroth moment:

$$\mathscr{D}_i\mathscr{D}_j \underset{(0)}{\mathcal{N}}^{ij}(u_1, u_0) = \underbrace{4\Delta m(u_1, u_0)}_{\text{difference of ``charges''}} - \overbrace{4\int_{u_0}^{u_1} \mathrm{d}u_2 \mathcal{F}(u_2)}^{\text{integral of ``flux''}}$$

- Key features:
  - "Charge": constant when $N_{ij} = 0$
  - "Flux": vanishes when $N_{ij} = 0$
- Can a similar type of formula be written for higher moments?

Angular momentum aspect

- Angular momentum aspect *not* a "charge" (from $R_{ui} = 0$):

$$\partial_u N_i = \mathscr{D}^j \underbrace{\left(m h_{ij} + \frac{1}{2} \mathscr{D}_{[i} \mathscr{D}^k C_{j]k} \right)}_{\equiv m_{ij}} + \underbrace{\frac{1}{4} (N^{jk} \mathscr{D}_j C_{ki} + 3 C_{ij} \mathscr{D}_k N^{jk})}_{\equiv \mathcal{F}_i}$$

  & $m_{ij} \neq 0$ when $N_{ij} = 0$!

- New charge (***not* the only choice!**):

$$\tilde{N}_i(u, \tilde{u}) \equiv N_i(u) - (u - \tilde{u}) \mathscr{D}^j m_{ij}(u)$$

$$\Downarrow$$

$$\begin{aligned} \partial_u \tilde{N}_i(u, \tilde{u}) &= \dot{N}_i(u) - \mathscr{D}^j m_{ij} - (u - \tilde{u}) \mathscr{D}^j \dot{m}_{ij}(u) \\ &= \underbrace{\mathcal{F}_i - (u - \tilde{u}) \mathscr{D}^j \dot{m}_{ij}(u)}_{= 0 \text{ when } N_{ij} = 0!} \end{aligned}$$

Recovery of the first moment

- $\partial_u m_{ij}$ gives the news:

$$\partial_u m_{ij} = \frac{1}{2}\,\mathrm{STF}(\mathscr{D}_i\mathscr{D}_k)N^k{}_j + \mathcal{F}h_{ij}$$

- Evolution equation for $\tilde{N}_i(u,\tilde{u})$:

$$\partial_u \tilde{N}_i(u,\tilde{u}) = \underbrace{\mathcal{F}_i(u) - (u-\tilde{u})\mathscr{D}_i\mathcal{F}(u)}_{\equiv \tilde{\mathcal{F}}_i(u,\tilde{u})} - \frac{1}{2}(u-\tilde{u})\mathscr{D}_k\,\mathrm{STF}(\mathscr{D}_i\mathscr{D}_j)N^{jk}$$

- Set $\tilde{u} = u_0$, integrate from u_0 to u_1:

$$\mathscr{D}_k\,\mathrm{STF}(\mathscr{D}_i\mathscr{D}_j)\underset{(1)}{\mathcal{N}}^{jk}(u_1,u_0) = 2\left[-\Delta\tilde{N}_i(u_1,u_0) + \int_{u_0}^{u_1}\mathrm{d}u_2\tilde{\mathcal{F}}_i(u_2,u_0)\right]$$

- Moment $= \Delta$("charge") $+ \int$ "flux"

A pattern for all evolution equations

- (Linear) evolution for $\underset{(n)}{\mathcal{E}}{}_{ij}$ (from $R_{ij} = 0$):

$$\partial_u \underset{(n)}{\mathcal{E}}{}_{ij} \simeq \begin{cases} \frac{1}{3}\,\mathrm{STF}\,\mathscr{D}_i N_j & n = 0 \\ \mathcal{D}_{n-1} \underset{(n-1)}{\mathcal{E}}{}_{ij} & n > 0 \end{cases}, \qquad \text{w/ angular operator } \mathcal{D}_n$$

while

$$\partial_u N_i \simeq \mathscr{D}^j m_{ij}, \quad \partial_u m_{ij} \simeq \frac{1}{2}\,\mathrm{STF}(\mathscr{D}_i \mathscr{D}_k) N^k{}_j$$

- Taking ∂_u eventually brings us to N_{ij}:

$$\underset{(n)}{\mathcal{E}}{}_{ij} \longrightarrow \underset{(n-1)}{\mathcal{E}}{}_{ij} \longrightarrow \cdots \longrightarrow \underset{(0)}{\mathcal{E}}{}_{ij} \longrightarrow N_i \longrightarrow m_{ij} \longrightarrow N_{ij}$$

A charge for higher moments

- Definition of the charge (**again, *not* unique!**):

$$\underset{(n)}{\tilde{\mathcal{E}}}{}_{ij}(u, \tilde{u}) \equiv \underset{(n)}{\mathcal{E}}{}_{ij}(u) + \sum_{p=1}^{n} \frac{(\tilde{u} - u)^p}{p!} \mathcal{D}_{n-1} \cdots \mathcal{D}_{n-p} \underset{(n-p)}{\mathcal{E}}{}_{ij}(u)$$
$$+ \frac{1}{3} \frac{(\tilde{u} - u)^{n+1}}{(n+1)!} \mathcal{D}_{n-1} \cdots \mathcal{D}_0 \, \mathrm{STF} \, \mathscr{D}_i \left[N_j(u) + \frac{\tilde{u} - u}{n+2} \mathscr{D}^k m_{jk}(u) \right]$$

- Sum in evolution equation "telescopes", each term cancelling the next:

$$\partial_u \underset{(n)}{\tilde{\mathcal{E}}}{}_{ij}(u, \tilde{u}) \simeq \frac{1}{6} \frac{(\tilde{u} - u)^{n+2}}{(n+2)!} \mathcal{D}_{n-1} \cdots \mathcal{D}_0 \, \mathrm{STF} \, \mathscr{D}_i \left[\mathscr{D}_l \, \mathrm{STF}(\mathscr{D}_j \mathscr{D}_k) N^{kl}(u) \right]$$

- Integrate to get moment of the news: moment $\simeq \Delta$("charge")

Some details of the full theory

- Last slides were linear theory, but *nonlinearly* you get:

$$\mathcal{D}_{n-1} \cdots \mathcal{D}_0 \, \mathrm{STF} \, \mathscr{D}_i \left[\mathscr{D}_l \, \mathrm{STF}(\mathscr{D}_j \mathscr{D}_k) \underset{(n+2)}{\mathcal{N}}{}^{kl}(u_1, u_0) \right]$$
$$= (-1)^n 6 \left[\Delta \underset{(n)}{\tilde{\mathcal{E}}}{}_{ij}(u_1, u_0) - \int_{u_0}^{u_1} \mathrm{d}u_2 \, \underset{(n)}{\tilde{\mathcal{F}}}{}_{ij}(u_2, u_0) \right]$$

- Note: flux involves things not in the waveform!

$$\underset{(n)}{\tilde{\mathcal{F}}}{}_{ij}(u_2, u_0) = \underbrace{\underset{(n)}{\tilde{\mathcal{F}}}{}^{\mathrm{rad}}_{ij}(u_2, u_0)}_{\ni C_{ij}, N_{ij}} + \underbrace{\underset{(n)}{\tilde{\mathcal{F}}}{}^{\mathrm{nonrad}}_{ij}(u_2, u_0)}_{\substack{\ni C_{ij}, N_{ij}, m, N_i, \\ \underset{(0)}{\mathcal{E}}{}_{ij}, \ldots, \underset{(n-1)}{\mathcal{E}}{}_{ij}}}$$

- For example, $\underset{(0)}{\tilde{\mathcal{F}}}{}^{\mathrm{nonrad}}_{ij}(u, \tilde{u}) = \frac{1}{2}(\tilde{u} - u) m(u) N_{ij}(u)$
- Issue *does not arise* for zeroth or first moment

Inverting angular operators

To reconstruct moments from expressions, expand in tensor harmonics $(T^{E,B}_{\ell m})_{ij}$:

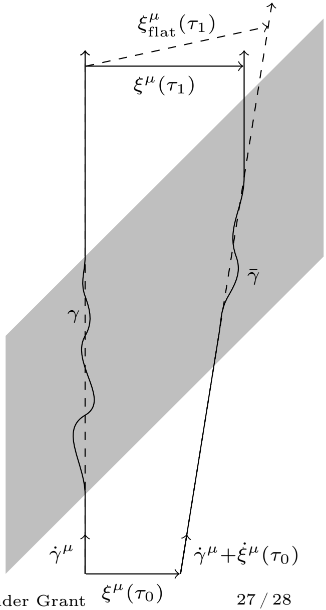
- $n = 0$: angular operators only annihilate magnetic harmonics
  $\implies$ can reconstruct electric part
  (magnetic part vanishes if stationary-to-stationary [Flanagan & Nichols, 2015])
- $n = 1, 2$: no $\ell \geq 2$ harmonics annihilated
  $\implies$ can fully reconstruct moments
- $n \geq 3$:
  - Expressions involve $\mathcal{D}_{n-3} \cdots \mathcal{D}_0$, where:

$$\mathcal{D}_p (T^{E,B}_{\ell m})_{ij} \propto \underbrace{[(p+2)(p+3) - \ell(\ell+1)]}_{\text{annihilates } \ell = p+2 \text{ harmonic}} (T^{E,B}_{\ell m})_{ij}$$

  - Can only reconstruct $\ell \geq n$ harmonics
  - Related to "linear" Newman-Penrose constants [N & P, 1968], [Laurent's talk]

Conclusions

- Persistent observables provide general framework for memory-like effects
- Curve deviation generalizes displacement memory, allowing for initial relative velocity and acceleration
- "Effective measurement" asymptotically: detection of non-zero moments of the news
- Computing moments involves increasingly subleading metric functions, along with "flux" terms, w/ caveats:
  - Flux not solely determined from waveform!
  - Only certain harmonics can be reconstructed!



Future work

- Use "consistency conditions" to test CCE
- Relation of source to $\underset{(n)}{\mathcal{E}}{}_{ij}$'s
  $\implies$ find $\Delta \underset{(n)}{\tilde{\mathcal{E}}}{}_{ij}(u_1, u_0)$, $\int_{u_0}^{u_1} \mathrm{d}u_2 \underset{(n)}{\tilde{\mathcal{F}}}{}_{ij}^{\text{nonrad}}(u_2, u_0)$
  [i.e., is Δ("charge") or $\int$ "flux" larger?]
- Can we measure non-zero moments of news ($n \geq 2$)?
- These moments describe $O(1/r)$ observables; what about $O(1/r^2)$:
  - Velocity/rotation (i.e., "holonomies")
  - Proper time [i.e., $O(\xi^2)$]